# An Explicit-Implicit Analysis Scheme in a General-Purpose FEA Environment

Abed M. Khaskia
Mallett Technology, Inc.
Laurel, Maryland
(301) 725-0060



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# Outline

- Introduction
- System of Equations  [Equations of Motion]
- Explicit Solution Scheme
- Implicit Scheme
- Mixed Explicit – Implicit
- Illustrative Case

# Introduction - Applications

- Vibration analysis
- Impact analysis. Crashworthiness, Drop test
- Rotating elements and machinery
- Earthquake analysis
- Explosives
- Metal Forming/stamping/rolling
- Random Vibration



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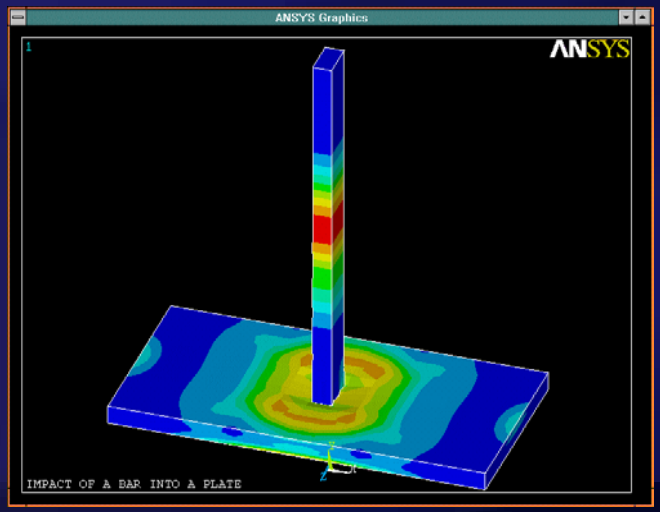
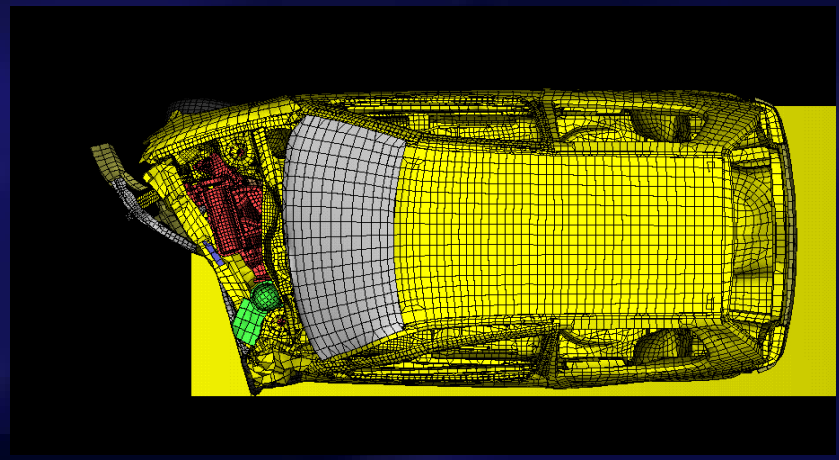
# Applications




Projectile Impact



Car Crash



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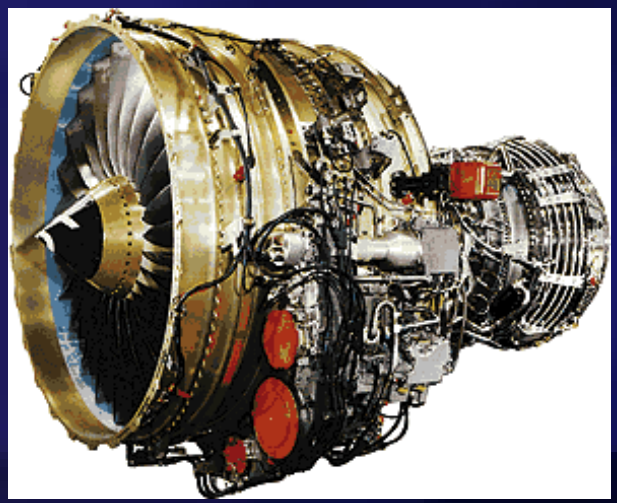
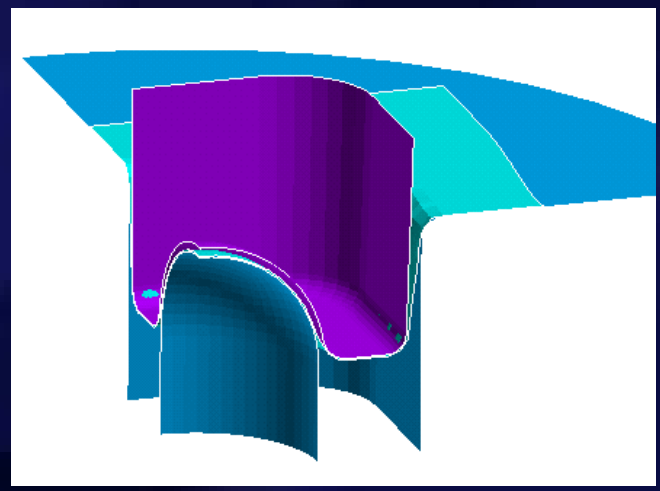
# Applications



Jet Engine Fan Containment



Metal Forming



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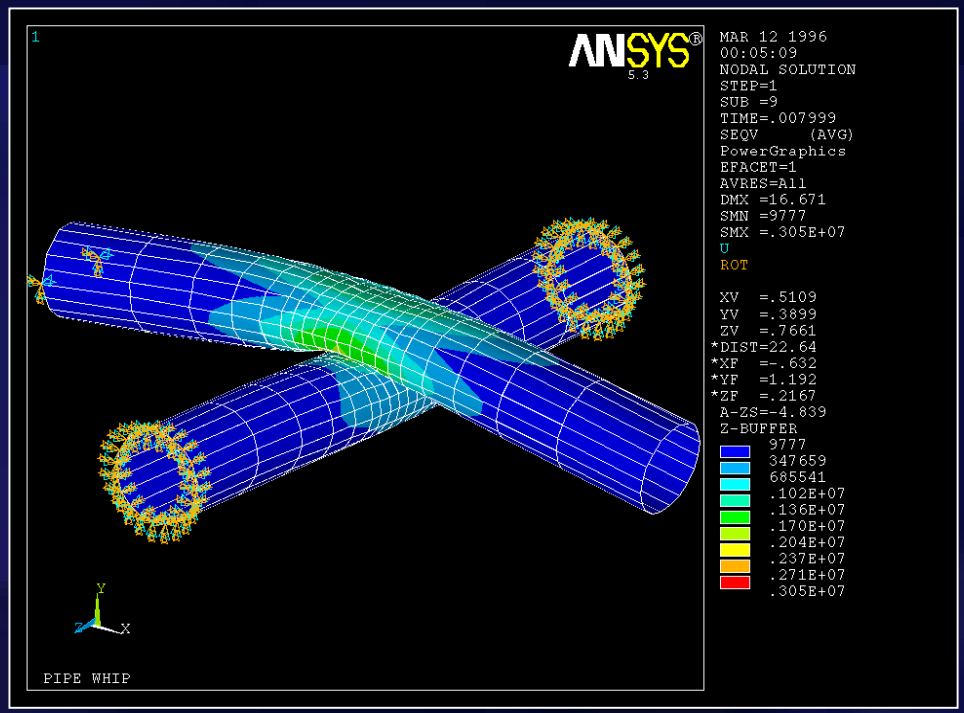
# Applications




Pipe Whip Problem



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# Introduction - Challenges

- Large systems
- Material and geometrical behavior
- Unknown material properties
- Loading and system boundary conditions
- Multiphysics and multiple domains
- Available testing and verifications issues
- Changing technologies in numerical analysis

# System of Equations

- General Equations of Motion

$$M\ddot{u}+C\dot{u}+Ku=F(t)$$

  - Solutions:
    - Implicit
    - Explicit
    - Mixed dictated by physics and numerical behavior

# Implicit Scheme

$$M\,\ddot{u} + C\,\dot{u} + K\,u = F(t)$$

Resulting Equations

$$M\ddot{u}_{t+\Delta t} + C\dot{u}_{t+\Delta t} + Ku_{t+\Delta t} = F(t)$$

Integration Scheme, Example, Newmark

$$u_{t+\Delta t} = u_t + \dot{u}_t\,\Delta t + [(1/2-\alpha)\ddot{u}_t + \alpha\ddot{u}_{t+\Delta t}]\,\Delta t^2$$

$$\dot{u}_{t+\Delta t} = \dot{u}_t + [(1-\delta)\ddot{u}_t + \delta\ddot{u}_{t+\Delta t}]\,\Delta t$$

Effectively Resulting in

$$u_{t+\Delta t} = K^{-1}\,F_{t+\Delta t}$$



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# Implicit Scheme

Nonlinear Case – Requires Newton-Raphson Iterations and Satisfying Equilibrium

$$K^T \delta u = \delta F$$

# Explicit Scheme

- No matrix conversion
- Computations of internal and external force vectors

$$\{a_t\} = M^{-1}\left(F_{ext} - F_{int}\right)$$

$$F_{int} = \Sigma\left(\int_{\Omega} B^T \sigma_n d\,\Omega + F_{etc}\right) + F_{cont}$$

$$v_{t+\Delta t/2} = v_{t-\Delta t/2} + a_t \Delta t_t$$

$$u_{t+\Delta t} = u_t + v_{t+\Delta t/2}\ \Delta t_{t+\Delta t/2}$$

- Critical Time Step

$$x_{t+\Delta t} = x_o + u_{t+\Delta t}$$

$$\Delta t \le \Delta t^{crit} = \frac{2}{\omega_{max}}$$

$$\omega_{max} = 2\frac{c}{l} \qquad \Delta t = \frac{l}{c} \qquad c = \sqrt{\frac{E}{\rho}}$$



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# Comparisons/Issues

- Stability
- Time step size
- Nonlinear effects
- Computations
- Convergence
- Mass matrices



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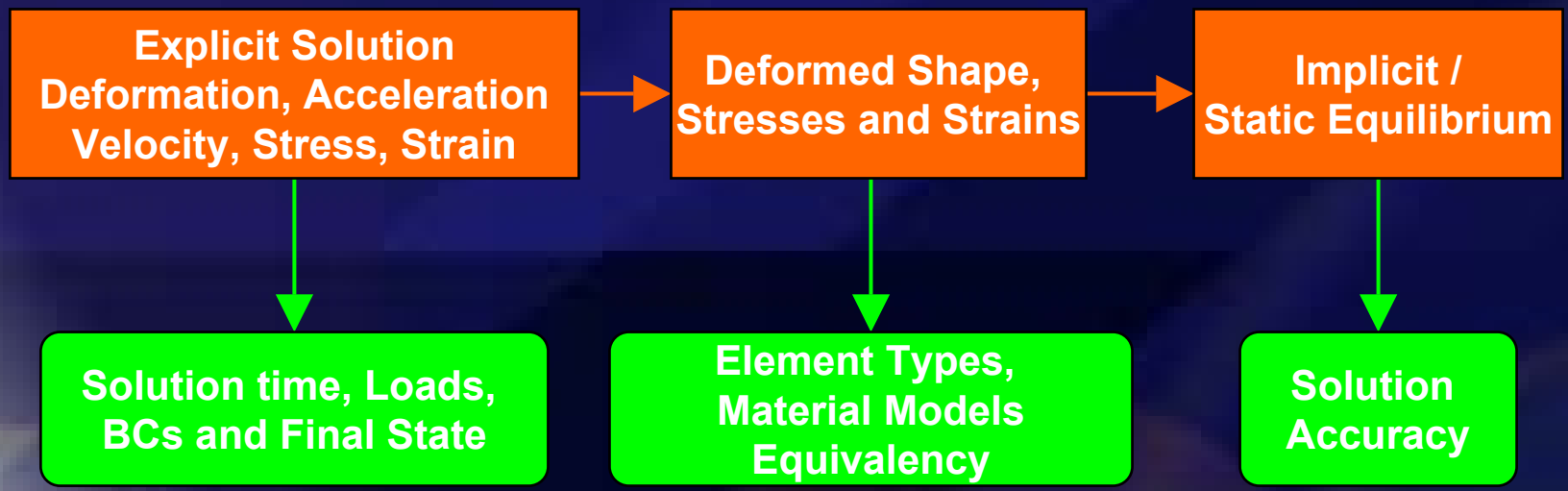
# Mixed Scheme / Explicit - Implicit

- Solution Steps

$$\{a_t\} = M^{-1}\left(F_{ext} - F_{int}\right)$$

$$F_{int} = \Sigma\left(\int_{\Omega} B^T \sigma_n d\,\Omega + F_{etc}\right) + F_{cont}$$

$$K^T \delta u = \delta F$$



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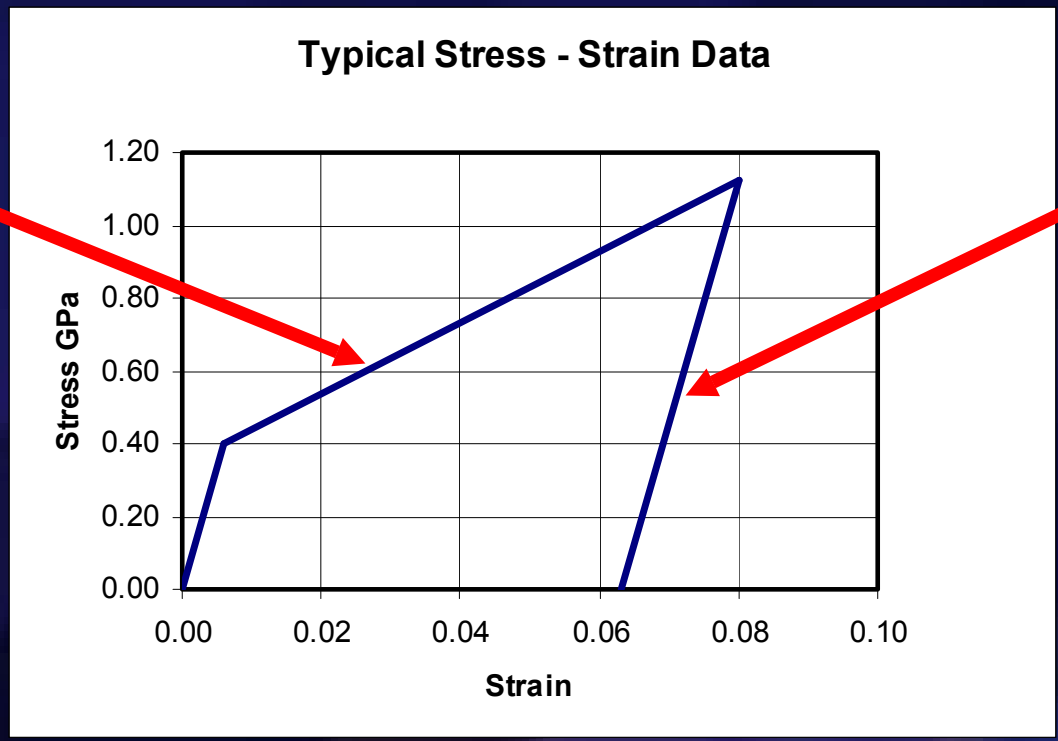
# Mixed Scheme / Explicit - Implicit

- Solution Process on Material Model

Explicit




Implicit



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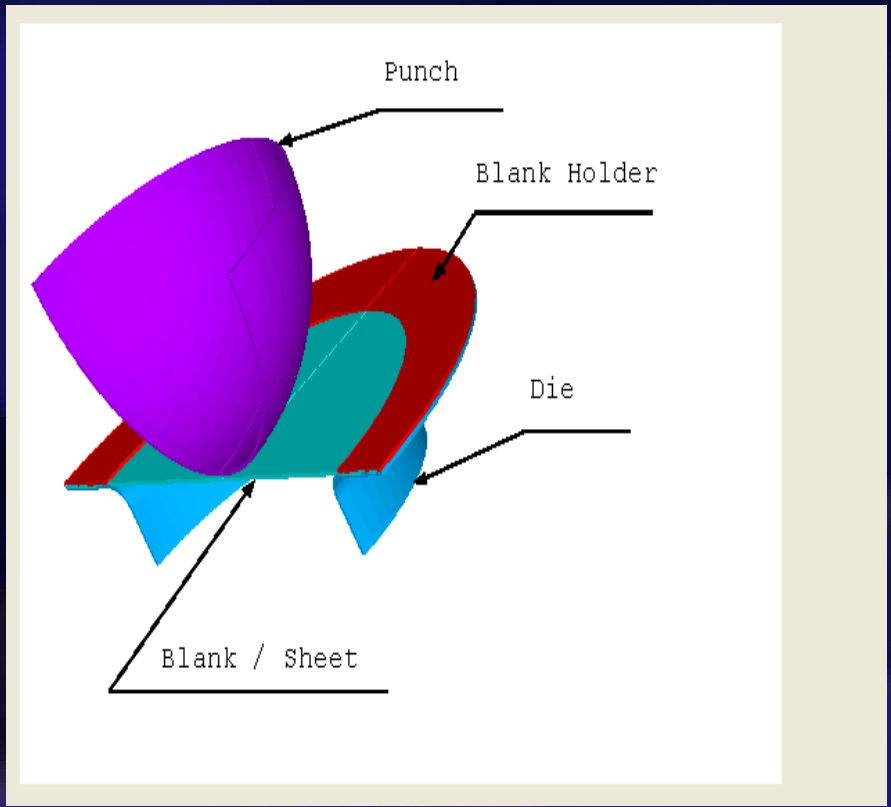
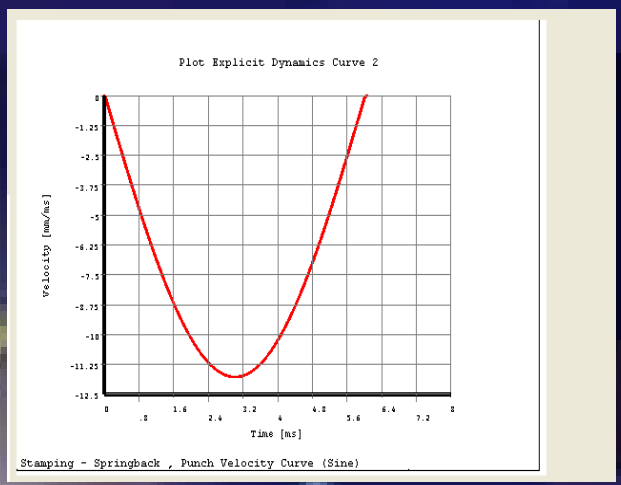
# Explicit to Implicit
## Case Study – Cup Stamping

- System Description and FE Model

- **Force is applied at blank holder**
- **Sinusoidal velocity is applied at punch**
- **Mass and stiffness damping**
- **Friction between components**
- **Mass Scaling**



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# Explicit to Implicit
## Case Study – Cup Stamping

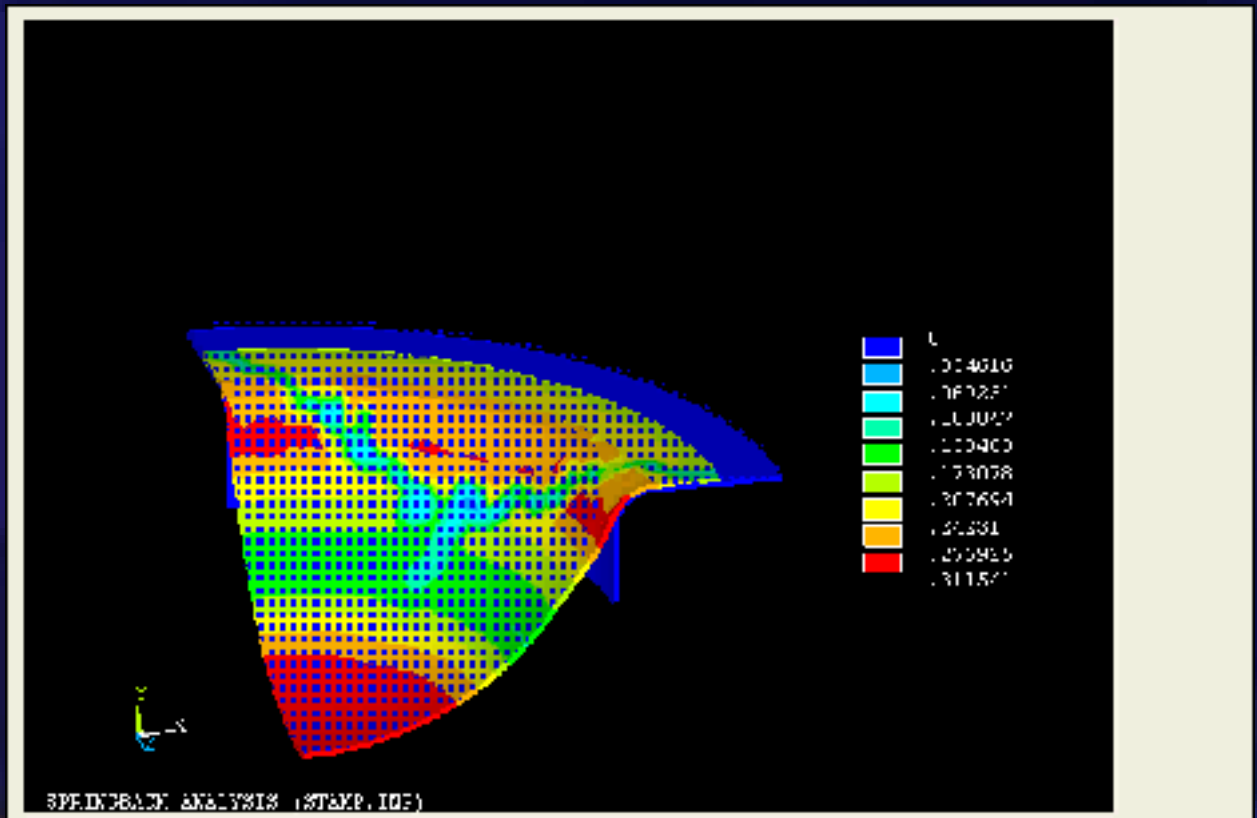
- Modeling Challenge
  - Mass scaling to speed solution
  - Solution accuracy and verifications
  - Damping
  - Friction effects
  - Element deformation and proper shape
  - Time point and process to go from explicit to implicit
  - Preventing Rigid body motion in implicit solution
  - Convergence of the nonlinear implicit solution

# Explicit to Implicit
## Case Study – Cup Stamping

- Results
  - Animation of process
  - Quality of solution – Hourglass energy check
  - Force applied by punch and blank velocity
  - Fluctuation in stress and strain data
  - Deformed shape and plastic strains at end of explicit
  - Spring back shape after implicit switch

# Explicit to Implicit
## Case Study – Cup Stamping

- Stamping Process



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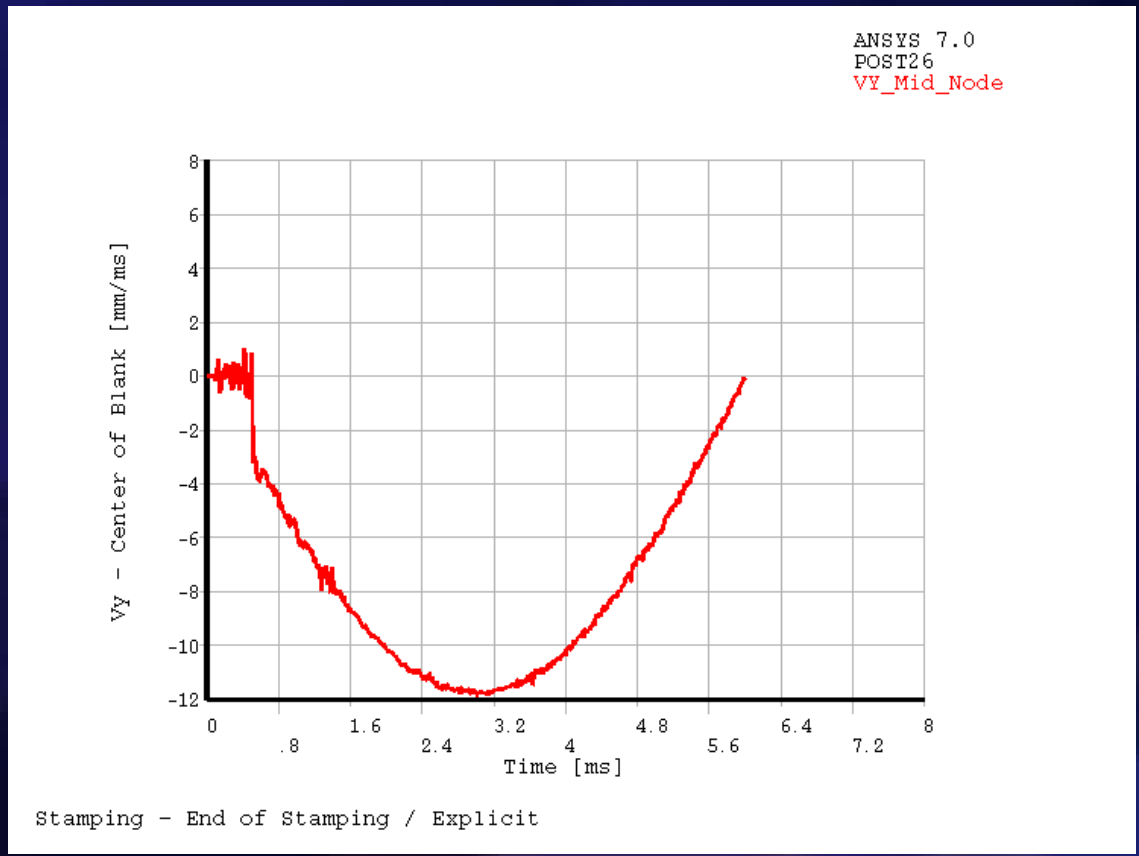
# Explicit to Implicit
## Case Study – Cup Stamping

- Blank Velocity



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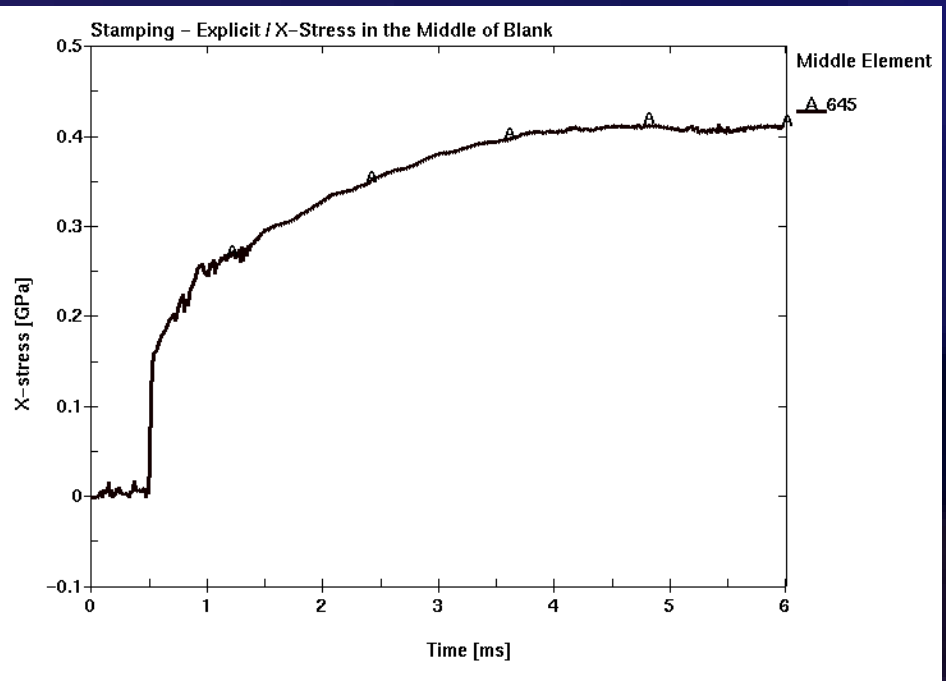
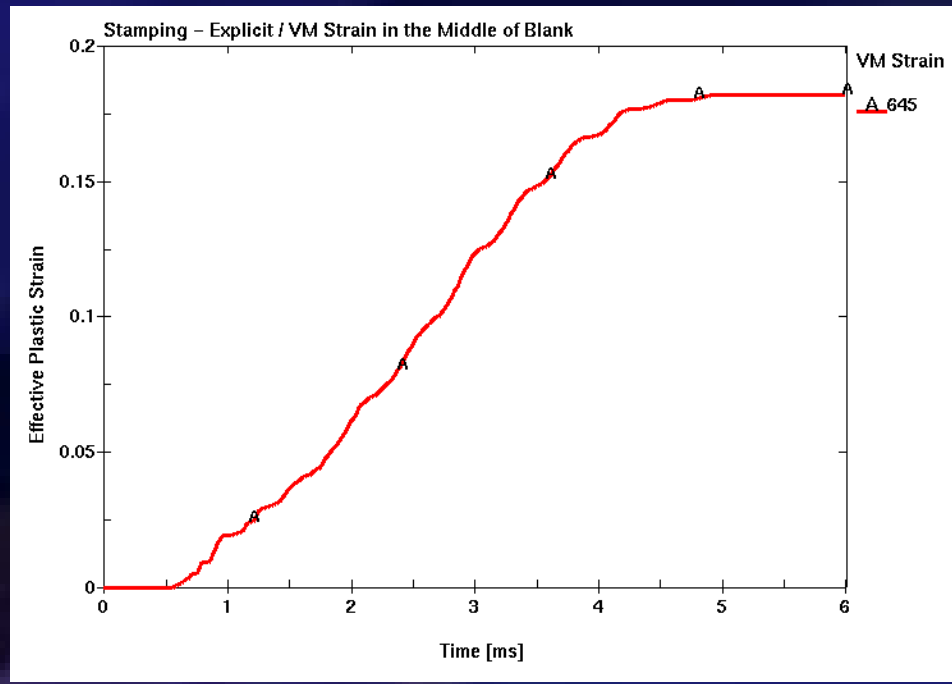
# Explicit to Implicit
## Case Study – Cup Stamping

- Stress and Strains



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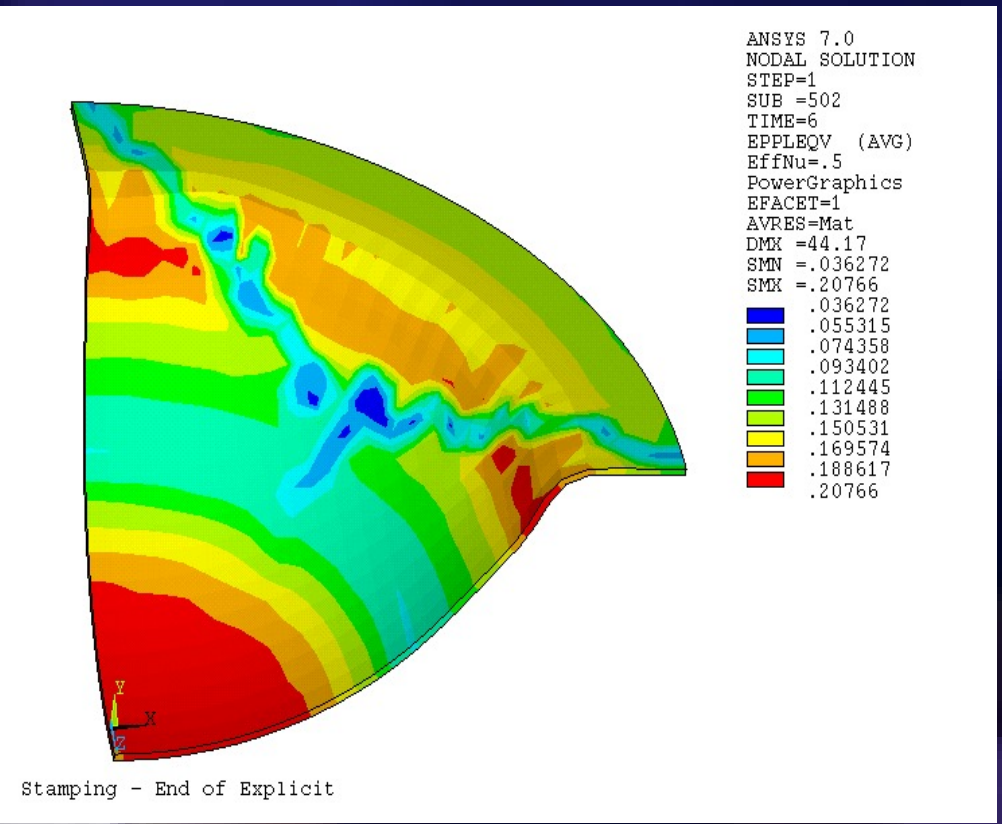
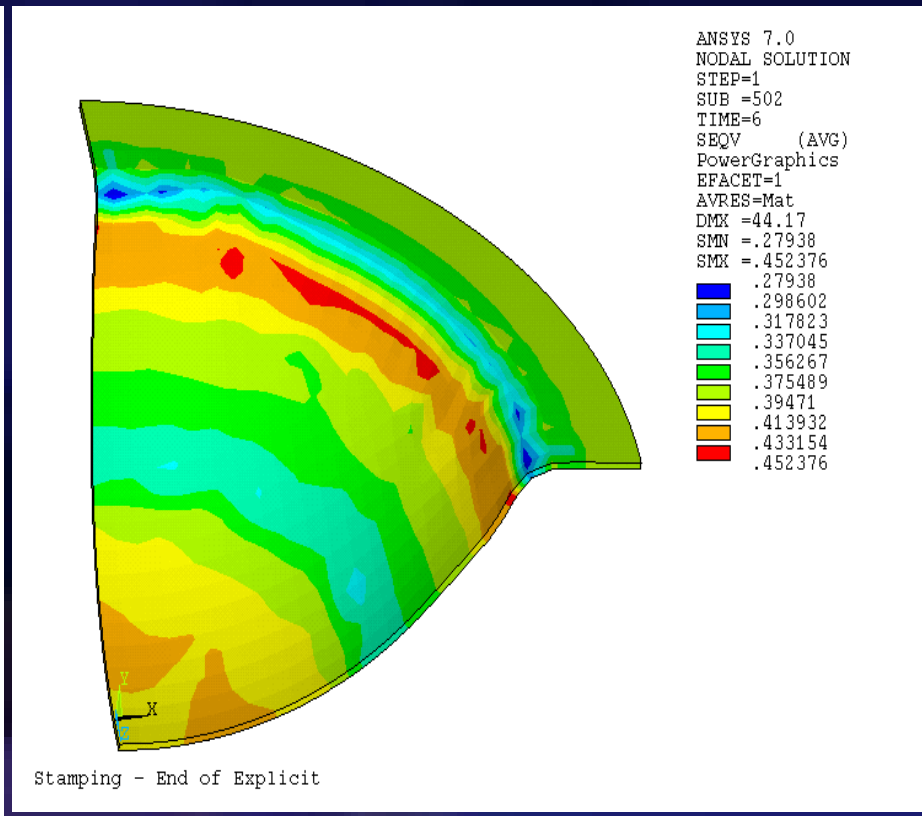
# Explicit to Implicit
## Case Study – Cup Stamping

- Deformed shape and plastic strains at end of explicit



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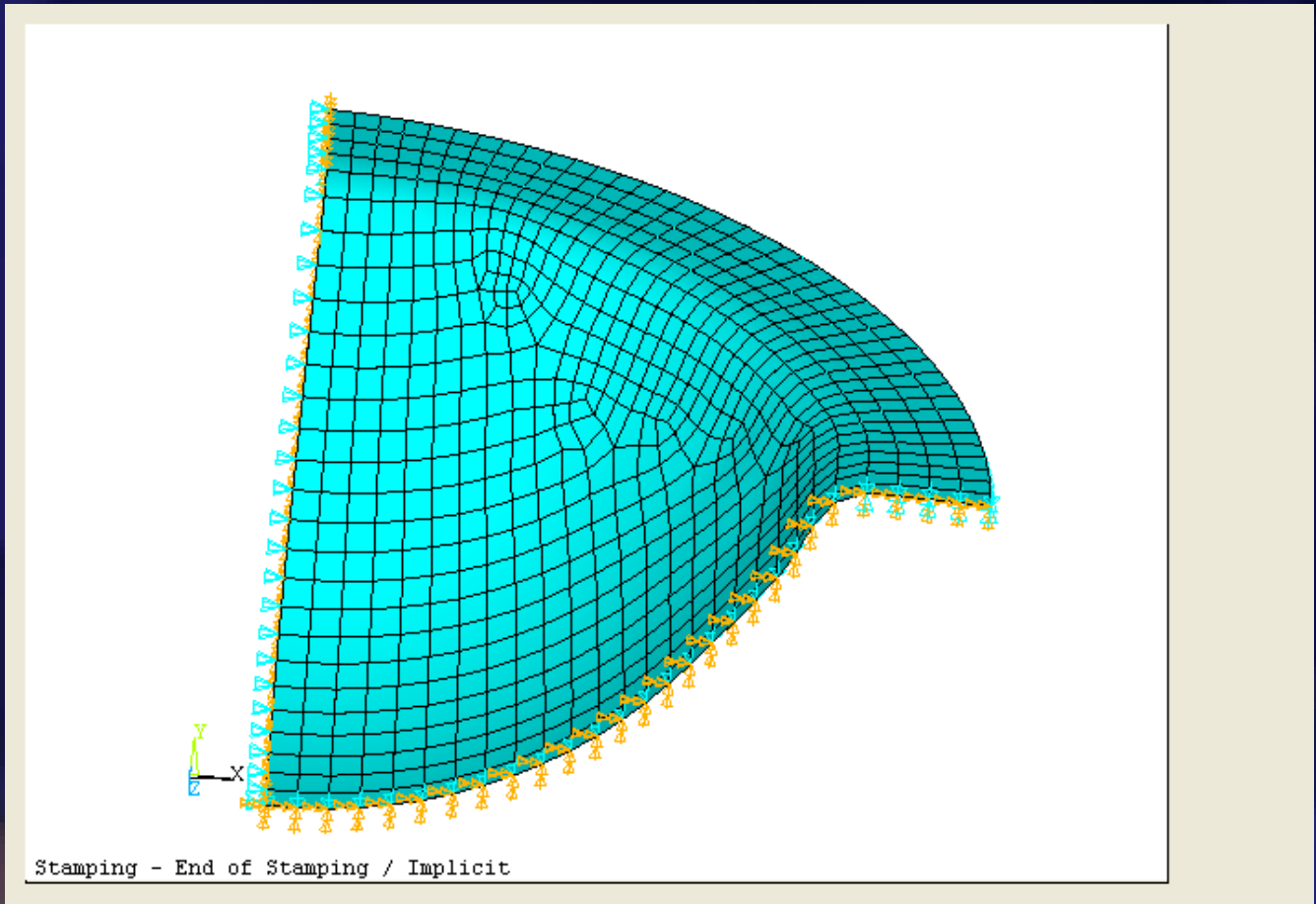
# Explicit to Implicit
## Case Study – Cup Stamping

- Implicit FE Model



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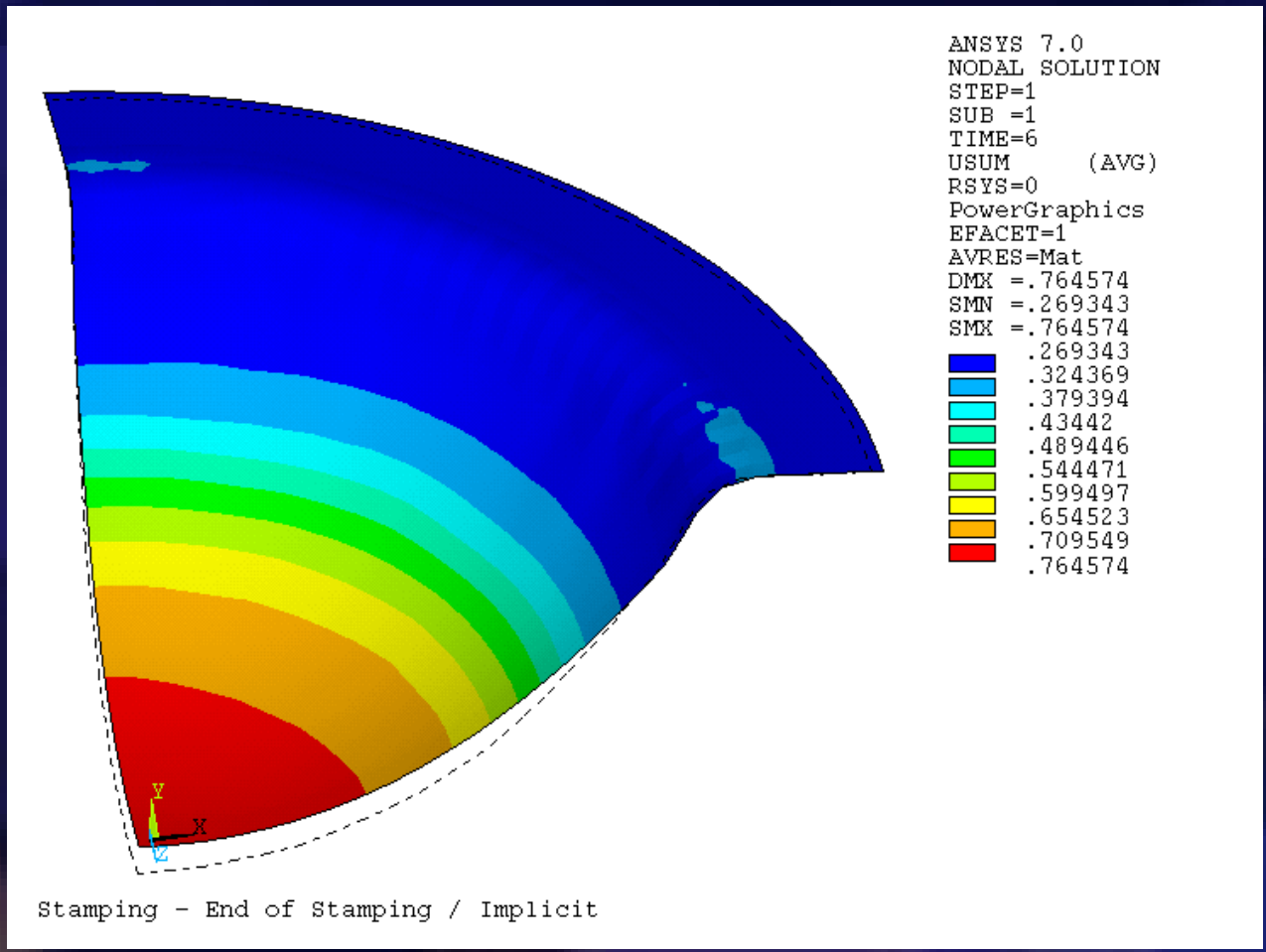
# Explicit to Implicit
## Case Study – Cup Stamping

- Spring back shape



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# Explicit to Implicit
## Case Study – Cup Stamping
## Conclusions

- Experience shows that explicit/implicit is less than 25% of implicit CPU for same application
- Implicit only is easier to validate and hence provides more confidence
- Rigid body constraints in implicit part and time location for switch
- Mass scaling and speed of process introduce simplifications
- Certain aspects of the explicit/implicit process could be automated
- Elements selections and compatibilities among them
- Data management is important as time scale has two different meanings
- The process is very promising for nonlinear applications as solvers will switch automatically between the two schemes based on solution behavior



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# Acknowledgments/References

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